Available: https://doi.org/10.54117/ijps.v2i2.14 Research article

IPS Journal of Physical Sciences
ISSN: 3115-5715
IPS J Phys Sci, 2(2): 81-89 (2025)
DOI: https://doi.org/10.54117/ijps.v2i2.14

Transforming Unreplicated Factorial Designs into Replicated Structures through Factor Projection

Ukeme P. Akra[1], Itoro T. Michael[1], Anthony A. Isaac[1], Andrew C. Etim[1] and Uwem A. Akpan[2]

[1]Department of Statistics, Akwa Ibom State University, Ikot Akpaden.
[2]Department of Statistics, Trinity Polytechnic, Mbak/Ifa.
*Corresponding author: ukemeakra@aksu.edu.ng

Abstract

In factorial experiments, unreplicated design is limited by the absence of error estimation, which complicates the identification of significant effects. This often leads to reliance on the sparsity-of-effects principle, where only a few main effects and lower-order interactions are considered meaningful, while most higher-order interactions are assumed negligible. To address this challenge, this study introduces a method for projecting unreplicated factorial designs into replicated design by reducing the number of factors and increase the number of replicates. This approach utilizes factorial effect estimation, normal probability plotting, and significance testing to identify influential factors. A full factorial design involving five binary factors (A, B, C, D, and E) was analyzed in an unreplicated 2^5 setup. The analysis indicated that factors A and E do not significantly affect the outcome, while AE interaction was minimal. However, factor B, and interactions AB, BE, and ABE shows significant effects. Based on these findings, the original 2^5 unreplicated design was projected into a 2^3 factorial design involving factors A, B, and E, including AE and BE interactions, with four replicates to enable error estimation. The results demonstrate that decreasing the number of factors (k) in the design enables an increase in the number of replicates, enhancing the reliability of inference through better error estimation.

Keywords: Projection, Unreplication, Replication, Factor, Factorial Design.

Article History
Received: 22 Oct 2025
Accepted: 04 Nov 2025
Published: 08 Nov 2025

Scan QR code to view•

How to cite this paper: Akra, U. P., Michael, I. T., Isaac, A. A., Etim, A. C., & Akpan, U. A. (2025). Transforming Unreplicated Factorial Designs into Replicated Structures through Factor Projection. *IPS Journal of Physical Sciences*, *2*(2), 81–89. https://doi.org/10.54117/ijps.v2i2.14

1. Introduction

In factorial designs, unreplicated factors do not allow for direct estimation of experimental error. To analyze such single-replicate designs, it is typically assumed that higher-order interactions are negligible. These assumptions enable the pooling of higher-order interaction mean squares to estimate the error term, reflecting the sparsity-of-effects principle where only a few main effects and lower-order interactions are likely to be significant, while most higher-order interactions are assumed to be negligible. A major challenge in unreplicated factorial experiments is the presence of outliers, which can significantly distort the estimation of model parameters. When outliers are present in the response values whether in a full or fractional factorial design the entire set of estimated effects may become biased. A single outlier resulting from measurement or recording error can compromise the reliability of all effect estimates. Therefore, assessing the influence of outliers is crucial to understanding their impact on model accuracy and interpretation (Lawson, 2008, Akpan et al., 2017). Another analytical challenge arises because pooling high-order interactions to estimate error may not always be appropriate in unreplicated designs. As such, considerable attention has been devoted to the objective analysis of unreplicated two-level and fractional factorial experiments. For a design with n runs, although n–1 effects (excluding the overall mean) can be estimated using contrasts, there are no degrees of freedom left to estimate the residual variance. This limitation prevents the use of conventional t-tests to identify statistically significant ("active") effects. In practice, a widely accepted method for identifying significant effects in unreplicated designs is the normal probability plot of contrasts, first proposed by Daniel (1959). This graphical technique allows researchers to visually distinguish active effects from those that are likely due to random variation. Notably, deterministic computer simulation experiments which cannot be replicated provide opportunities to apply such analysis methods (Willers & Vinyard, 2019).

Several prior studies (Akra et al., 2024; Akra & Edet, 2017; Akra et al., 2025) have successfully employed factorial designs in different domains. These include mixed-level factorial designs for evaluating the effect of inorganic fertilizers on crop yield, confounded 2^k factorial designs for assessing organic manure applications, and mixed-level designs to address uncertainty in retail business planning. In unreplicated two-factor experiments, selected contrasts can be analyzed using normal probability plots. This approach allows researchers to identify which main effects and interactions are significantly different from zero. Each contrast corresponds to a specific effect; if it is statistically significant, the associated main effect or interaction is inferred to be non-zero. When the response variable is multivariate and at least one treatment is replicated, Multivariate Analysis of Variance (MANOVA) is commonly used (Meixi, 2014). However, for designs with only one replicate per treatment, analytical tools are limited. In such cases, the model becomes a multivariate extension of its univariate counterpart, where both main effects and interactions are represented as parameter vectors. Tukey (1949) extended analysis methods for unreplicated two-factor experiments, followed by the development of multivariate contrast techniques suitable for factorial designs. The appropriate use of unreplicated designs in agricultural and biological research can help minimize costs and reduce dependence on in vivo testing (Carla & Linda, 2016). Furthermore, the Lenth method (Kenny & Hamada, 2000) offers an objective approach to analyzing unreplicated factorial designs, eliminating the subjectivity involved in interpreting half-normal plots. Montgomery (2019) discussed unreplicated factorial designs, challenges in estimating experimental error, and methods for introducing replication or using techniques such as pooling, foldover designs, and adding center points to achieve replication. Wu and Hamada (2009) explains how replication enhances precision and reliability, and offers approaches for augmenting an existing factorial design to include replicates without losing orthgonality.

This study aims to project an unreplicated factorial design onto a replicated factorial design by focusing on a reduced number of significant factors. The objective is to identify the non-zero (i.e., significant) effects and assess the improvement in model accuracy with replication.

2. Methodology

Consider a factorial design involving K factors, each at two levels. In this study, five factors; A, B, C, D, and E—were examined, resulting in a 2^5 full factorial design with 32 treatment combinations. The standard order of these combinations is as follows: (1), a, b, ab, c, ac, bc, abc, d, ad, bd, abd, cd, acd, bcd, abcd, e, ae, abe, ce, ace, bce, abce, de, ade, bde, abde, cde, acde, bcde, abcde. There are three common notations for representing runs in a 2^5 design: The geometric notation "+" and "–" indicates high and low levels used in this study is illustrated in Table 1.

Table 1 a: Algebraic signs for calculating effects of 2^5 Design

T C	A	B	AB	C	AC	BC	ABC	D	AD	BD	ABD	CD	ACD	BCD	ABCD	E
(1)	+	-	+	-	+	+	-	-	+	+	-	+	-	-	+	-
a	+	-	-	-	-	+	+	-	-	+	+	+	+	-	-	-
b	-	+	-	-	+	-	+	-	+	-	+	+	-	+	-	-
ab	+	+	+	-	-	-	-	-	-	-	-	+	+	+	+	-
c	-	-	+	+	-	-	+	-	+	+	-	-	+	+	-	-
ac	+	-	-	+	+	-	-	-	-	+	+	-	-	+	+	-
bc	-	+	-	+	-	+	-	-	+	-	+	-	+	-	+	-
abc	+	+	+	+	+	+	+	-	-	-	-	-	-	-	-	-
d	-	-	+	-	+	+	-	+	-	-	+	-	+	+	-	-
ad	+	-	-	-	-	+	+	+	+	-	-	-	-	+	+	-
bd	-	+	-	-	+	-	+	+	-	+	-	-	+	-	+	-
abd	+	+	+	-	-	-	-	+	+	+	+	-	-	-	-	-
cd	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+	-
acd	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-	-
bcd	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-	-
abcd	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	-
e	-	-	+	-	+	+	+	-	+	+	-	+	-	-	+	+
ae	+	-	-	-	-	+	+	-	-	+	+	+	+	-	-	+
be	-	+	-	-	+	-	+	-	+	-	+	+	-	+	-	+
abe	+	+	+	-	-	-	-	-	-	-	-	+	+	+	+	+
ce	-	-	+	+	-	-	+	-	+	+	-	-	+	+	-	+
ace	+	-	-	+	+	-	-	-	-	+	+	-	-	+	+	+
bce	-	+	-	+	-	+	-	-	+	-	+	-	+	-	+	+
abce	+	+	+	+	+	+	+	-	-	-	-	-	-	-	-	+
de	-	-	+	-	+	+	-	+	-	-	+	-	+	+	-	+
ade	+	-	-	-	-	+	+	+	+	-	-	-	-	+	+	+
bde	-	+	-	-	+	-	+	+	-	+	-	-	+	-	+	+
abde	+	+	+	-	-	-	-	+	+	+	+	-	-	-	-	+
cde	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+	+
acde	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+
bcde	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-	+
abcde	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+

Table 1 b: Algebraic signs for calculating effects of 2^5 Design

TC	AE	BE	ABE	CE	ACE	BCE	ABCE	DE	ADE	BDE	ABDE	CDE	ACDE	BCDE	ABCDE
(1)	+	+	-	+	-	-	+	+	-	-	+	-	+	+	-
a	-	+	+	+	+	-	-	+	+	-	-	-	-	+	+
b	+	-	+	+	-	+	-	+	-	+	-	-	+	-	+
ab	-	-	-	+	+	+	+	+	+	+	+	-	-	-	-
c	+	+	-	-	+	+	-	+	-	-	+	+	-	-	+
ac	-	+	+	-	-	+	+	+	+	-	-	+	+	-	-
bc	+	-	+	-	+	-	+	+	-	+	-	+	-	+	-
abc	-	-	-	-	-	-	-	+	+	+	+	+	+	+	+
d	+	+	-	+	-	-	+	-	+	+	-	+	-	-	+
ad	-	+	+	+	+	-	-	-	-	+	+	+	+	-	-
bd	+	-	+	+	-	+	-	-	+	-	+	+	-	+	-
abd	-	-	-	+	+	+	+	-	-	-	-	+	+	+	+
cd	+	+	-	-	+	+	-	-	+	+	-	-	+	+	-
acd	-	+	+	-	-	+	+	-	-	+	+	-	-	+	+
bcd	+	-	+	-	+	-	+	-	+	-	+	-	+	-	+
abcd	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
e	-	-	+	-	+	+	-	-	+	+	-	+	-	-	+
ae	+	-	-	-	-	+	+	-	-	+	+	+	+	-	-
be	-	+	-	-	+	-	+	-	+	-	-	+	-	+	-
abe	+	+	+	-	-	-	-	-	-	-	-	+	+	+	+
ce	-	-	+	+	-	-	+	-	+	+	-	-	+	+	-
ace	+	-	-	+	+	-	-	-	-	+	+	-	-	+	+
bce	-	+	-	+	-	+	-	-	+	-	+	-	+	-	+
abce	+	+	+	+	+	+	+	-	-	-	-	-	-	-	-
de	-	-	+	-	+	+	-	+	-	-	+	-	+	+	-
ade	+	-	-	-	-	+	+	+	+	-	-	-	-	+	+
bde	-	+	-	-	+	-	+	+	-	+	-	-	+	-	+
abde	+	+	+	-	-	-	-	+	+	+	+	-	-	-	-
cde	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+
acde	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-
bcde	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-
abcde	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+

2.1 Main Effects and Interactions

The main effects and interactions are estimated as follows using Table 1a and 1b

$$A = \frac{1}{16n}\begin{bmatrix} a + ab + ac + abc + ad + abd + acd + abcd + ae + abe + ace \\ +abce + ade + abde + acde + abcde - (1) - b - c - bc - d \\ -bd - cd - bcd - e - be - ce - bce - de - bde - cde - bcde \end{bmatrix} \quad (1)$$

$$B = \frac{1}{16n}\begin{bmatrix} b + ab + bc + abc + bd + abd + bcd + abcd + be + abe + bce \\ +abce + bde + abde + bcde + abcde - (1) - a - c - ac - d - \\ ad - cd - acd - e - ae - ce - ace - de - ade - cde - acde \end{bmatrix} \quad (2)$$

$$C = \frac{1}{16n}\begin{bmatrix} c + ac + bc + abc + cd + acd + bcd + abcd + ce + ace + bce \\ +abce + cde + acde + bcde + abcde - (1) - a - b - ab - d \\ -ad - bd - abd - e - ae - be - abe - de - ade - bde - abde \end{bmatrix} \quad (3)$$

$$D = \frac{1}{16n}\begin{bmatrix} d + ad + bd + abd + cd + acd + bcd + abcd + de + ade + bde \\ +abde + cde + acde + bcde + abcde - (1) - a - b - ab - c - \\ ac - bc - abc - d - ad - bd - abd - cd - acd - bcd - abcd \end{bmatrix} \quad (4)$$

$$E = \frac{1}{16n}\begin{bmatrix} e + ae + be + abe + e + ace + bce + abce + de + ade + bde + \\ abde + cde + acde + bcde + abcde - (1) - a - b - ab - c - \\ ac - bc - abc - d - ad - bd - abd - cd - acd - bcd - abcd \end{bmatrix} \quad (5)$$

$$AB = \frac{1}{16n}\begin{bmatrix} abcde - bcde - acde + cde + abde - bde - ade + de + abce - \\ bce - ace + ce + abe - be - ae + e + abcd - bcd - acd + cd \\ +abd - bd - ad + d + abc - bc - ac + c + ab - b - a - (1) \end{bmatrix} \quad (6)$$

$$AC = \frac{1}{16n}\begin{bmatrix} (1) - a + b - ab - c + ac - bc + abc + d - ad + bd - abd \\ -cd + acd - bcd + abcd + e - ae + be - abe - ce + ace - bce \\ +abce + de - ade + bde - abde - cde + acde - bcde + abcde \end{bmatrix} \quad (7)$$

$$AD = \frac{1}{16n}\begin{bmatrix} (1) - a + b - ab + c - ac + bc - abc - d + ad - bd + abd \\ -cd + acd - bcd + abcd + e - ae + be - abe + ce - ace + bce \\ -abce - de + ade - bde + abde - cde + acde - bcde + abcde \end{bmatrix} \quad (8)$$

$$AE = \frac{1}{16n}\begin{bmatrix} (1) - a + b - ab + c - ac + bc - abc + d - ad + bd - abd \\ +cd - acd + bcd - abcd - e + ae - be + abe - ce + ace - bce \\ +abce - de + ade - bde + abde - cde + acde - bcde - abcde \end{bmatrix} \quad (9)$$

$$BC = \frac{1}{16n}\begin{bmatrix} (1) + a - b - ab - c - ac + bc + abc + d + ad - bd - abd - cd \\ -acd + bcd + bcd + abcd + e + ae - be - abe - ce - ace + bce \\ +abce + de + ade - bde - abde - cde - acde + bcde + abcde \end{bmatrix} \quad (10)$$

$$BD = \frac{1}{16n}\begin{bmatrix} (1) + a - b - ab + c + ac - bc - abc - d - ad + bd + abd \\ -cd - acd + bcd + abcd + e + ae - be - abe + ce + ace - bce - \\ abce - de - ade + bde + abde - cde - acde + bcde + abcde \end{bmatrix} \quad (11)$$

$$BE = \frac{1}{16n}\begin{bmatrix} (1) + a - b - ab + c + ac - bc - abc + d + ad - bd - abd \\ +cd + acd - bcd - abcd - e - ae + be + abe - ce - ace + bce \\ +abce - de - ade + bde + abde - cde - acde + bcde + abcde \end{bmatrix} \quad (12)$$

$$CD = \frac{1}{16n}\begin{bmatrix} (1) + a + b + ab - c - ac - bc - abc - d - ad - bd - abd \\ +cd + acd + bcd + abcd + e + be + abe - ce - ace - bce \\ -abce - de - ade - abde + cde + acde + bcde + abcde \end{bmatrix} \quad (13)$$

$$CE = \frac{1}{16n}\begin{bmatrix} (1) + a + b + ab - c - ac - bc - abc + d + ad + bd + abd - \\ cd - acd - bcd - abcd - e - ae - be - abe + ce + ace + bce \\ +abce - de - ade - abd - abde + cde + acde + bcde + abcde \end{bmatrix} \quad (14)$$

$$DE = \frac{1}{16n}\begin{bmatrix} (1) + a + b + ab + c + ac + bc + abc - d - ad - bd - abd - \\ cd - acd - bcd - abcd - e - ae - be - abe - ce - ace - bce \\ -abce + de + ade + bde + abde + cde + acde + bcde + abcde \end{bmatrix} \quad (15)$$

$$ABC = \frac{1}{16n}\begin{bmatrix} abce - bcde - acde + cde - abde + bde + ade - de + abce \\ -bce - ace - ce - abe - be + ae + e \;\; + abcd - bcd - acd + cd \\ -abd + bd + ad - d + abc - bc - ac + c - ab + b + a - (1) \end{bmatrix} \quad (16)$$

$$ABD = \frac{1}{16n}\begin{bmatrix} abcde - bcde - acde + cda + abde - bde - ade + de - abce \\ +bce + ace - ce - abe + be + ae - e + abcd - bcd - acd + cd \\ +abd - bd - ad + d - abc + bc + ac - c - ab + b + a - (1) \end{bmatrix} \quad (17)$$

$$ACD = \frac{1}{16n}\begin{bmatrix} abcde - bcde + acde - cde - abde + bde - ade + de - abce \\ +bce - ace + ce + abe - be + ae - e + abcd - bde + acd - ce \\ -abd + bd - ad + d - abc - bc + ac + c + ab + b - a - (1) \end{bmatrix} \quad (18)$$

$$BCD = \frac{1}{16n}\begin{bmatrix} abcde + bcde - acde - cde - abde - bde + ade + de - abce \\ -bce + ace + ce + abe + be - ae - e + abcd + bcd - acd - cd \\ -abd - bd + ad + d - abc - bc + ac + c + ab + b - a - (1) \end{bmatrix} \quad (19)$$

$$ABE = \frac{1}{16n}\begin{bmatrix} abcde - bcde - acde + cde + abde - bde - ade + de + abce \\ -bce - ace + ce + abe - be - ae + e - abcd + bcd + acd - cd \\ -abd + bd + ad - d - abc + bc + ac - c - ab + b + a - (1) \end{bmatrix} \quad (20)$$

$$ACE = \frac{1}{16n}\begin{bmatrix} abcde - bcde + acde - cde - abde + bde - ade + de + abce \\ -bce + ace - ce - abe + be - ae + e - abcd + bcd - acd + cd \\ +abd - bd + ad - d - abc + bc - ac + c + ab - b + a - (1) \end{bmatrix} \quad (21)$$

$$BCE = \frac{1}{16n}\begin{bmatrix} abcde + bcde - acde - cde - abde - bde + ade + de \\ +abce + bce - ace - ce - abe - be + ae + e - abcd - bcd + acd \\ +cd + abd + bd - ad - d - abc - bc + ac + c + ab + b - a - (1) \end{bmatrix} \quad (22)$$

$$ADE = \frac{1}{16n}\begin{bmatrix} abcde - bcde + acde - cde + abde - bde + ade - de - abce \\ +bce - ace + ce - abe + be - ae + e - abcd + bcd - acd + cd \\ -abd + bd - ad + d + abc - bc + ac - c + ab - b + a - (1) \end{bmatrix} \quad (23)$$

$$BDE = \frac{1}{16n}\begin{bmatrix} abcde + bcde - acde - cde + abde + bde - ade - de - abce \\ -bce + ace + ce - abe - be + ae + e - abcd - bcd + acd + cd \\ -abd - bd + ad + d + abc + bc - ac - c + ab + b - a - (1) \end{bmatrix} \quad (24)$$

$$CDE = \frac{1}{16n}\begin{bmatrix} abcde + bcde + acde + cde - abde - bde - ade - de - abce - \\ bce - ace - ce + bce + be + ae + e - abcd - bcd - acd - cd \\ +abd + bd + ad + d + abc + bc + ac + c - ab - b - a - (1) \end{bmatrix} \quad (25)$$

$$ABCD = \frac{1}{16n}\begin{bmatrix} (1) - a - b + ab - c + ac + bc - abc - d + ad + bd - abd + \\ cd - acd - bcd + abcd + e - ae - be + abe - ce + ace + bce \\ -abce - de + ade + bde - abde + cde - acde - bcde + abcde \end{bmatrix} \quad (26)$$

$$ABCE = \frac{1}{16n}\begin{bmatrix} (1) - a - b + ab - c + ac + bc - abc + d - ad - bd + abd - \\ cd + acd + bcd - abcd - e + ae + be - abe + ce - ace - bce \\ +abce - de + ade + bde - abde + cde - acde - bcde + abcde \end{bmatrix} \quad (27)$$

$$ABDE = \frac{1}{16n}\begin{bmatrix} (1) - a - b + ab + c - ac - bc + abc - d + ad + bd - abd - \\ cd + acd + bcd - abcd - e + ae + be - abe - ce + ace + bce \\ -abce + de - ade - bde + abde + cde - acde - bcde + abcde \end{bmatrix} \quad (28)$$

$$ACDE = \frac{1}{16n}\begin{bmatrix} (1) - a + b - ab - c + ac - bc + abc - d + ad - bd + abd + cd \\ -acd + bcd - abcd - e + ae - be + abe + ce - ace + bce \\ -abce + de - ade + bde - abde - cde + acde - bcde + abcde \end{bmatrix} \quad (29)$$

$$BCDE = \frac{1}{16n}\begin{bmatrix} (1) + a - b - ab - c - ac + bc + abc - d - ad + bd + abd + cd \\ +acd - bcd - abcd - abcd - e - ae + be + abe + ce + ace - bce \\ -abce + de + ade - bde - abde - cde - acde + bcde + abcde \end{bmatrix} \quad (30)$$

$$ABCDE = \frac{1}{16n}\begin{bmatrix} abcde - bcde - acde + cde - abde + bde + ade - de - abce \\ +bce + ace - ce + abe - be - ae + e - abcd + bcd + acd - cd \\ +abd - bd - ad + d + abc - bc - ac + c - ab + b + a - (1) \end{bmatrix} \quad (31)$$

3. Results

3.1 The analysis of variance result for 2^5 unreplicated factorial design is shown in Table 2, and the data used for the analysis is displayed in appendix 1

Table 2: Analysis of variance for 2^5 unreplicated factorial design

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	31	11664.0	376.26	*	*
Linear	5	4166.2	833.23	*	*
A	1	2502.8	2502.78	*	*
B	1	770.3	770.28	*	*
C	1	19.5	19.53	*	*
D	1	22.8	22.78	*	*
E	1	850.8	850.78	*	*
2-Way Interactions	10	2281.8	228.18	*	*
A*B	1	26.3	26.28	*	*
A*C	1	0.3	0.28	*	*
A*D	1	457.5	457.53	*	*
A*E	1	13.8	13.78	*	*
B*C	1	116.3	116.28	*	*
B*D	1	38.3	38.28	*	*
B*E	1	5.3	5.28	*	*
C*D	1	790.0	790.03	*	*
C*E	1	140.3	140.28	*	*
D*E	1	693.8	693.78	*	*
3-Way Interactions	10	1270.8	127.08	*	*
A*B*C	1	185.3	185.28	*	*
A*B*D	1	87.8	87.78	*	*
A*B*E	1	34.0	34.03	*	*
A*C*D	1	3.8	3.78	*	*
A*C*E	1	0.3	0.28	*	*
A*D*E	1	11.3	11.28	*	*
B*C*D	1	810.0	810.03	*	*
B*C*E	1	38.3	38.28	*	*
B*D*E	1	52.5	52.53	*	*
C*D*E	1	47.5	47.53	*	*
4-Way Interactions	5	3897.7	779.53	*	*
A*B*C*D	1	3676.5	3676.53	*	*
A*B*C*E	1	132.0	132.03	*	*
A*B*D*E	1	81.3	81.28	*	*
A*C*D*E	1	7.0	7.03	*	*
B*C*D*E	1	0.8	0.78	*	*
5-Way Interactions	1	47.5	47.53	*	*

A*B*C*D*E	1	47.5	47.53	*	*
Error	0	*	*		
Total	31	11664.0			

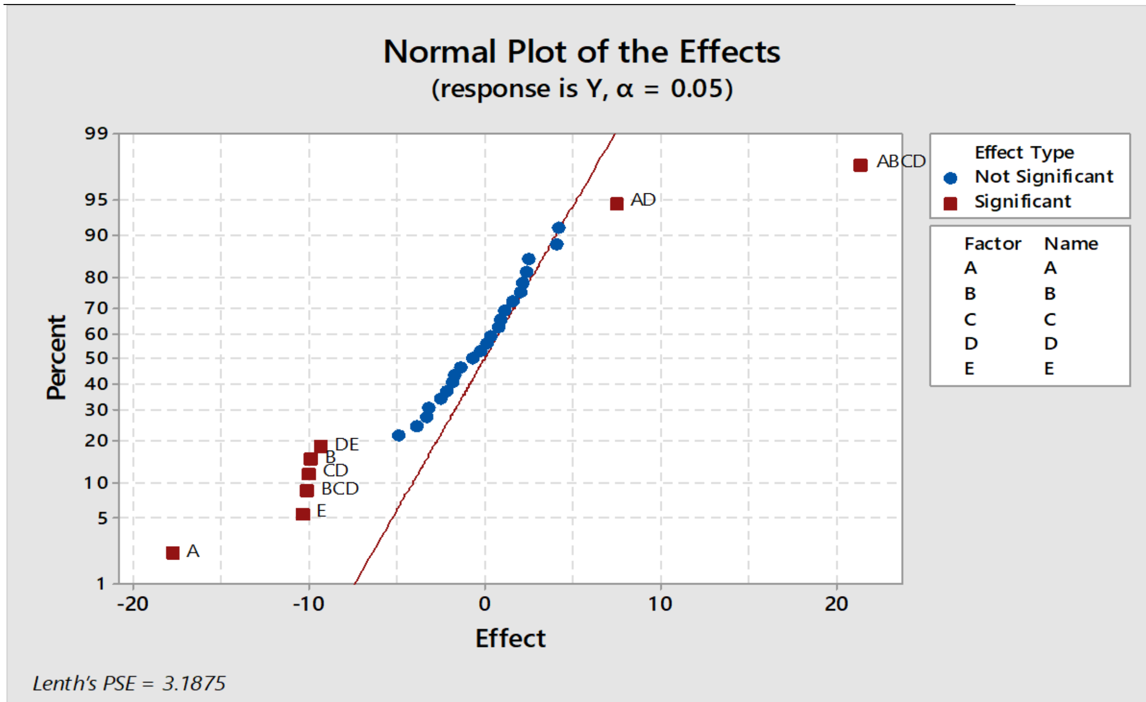

Figure 1: Normal probability plot showing the significance of factors

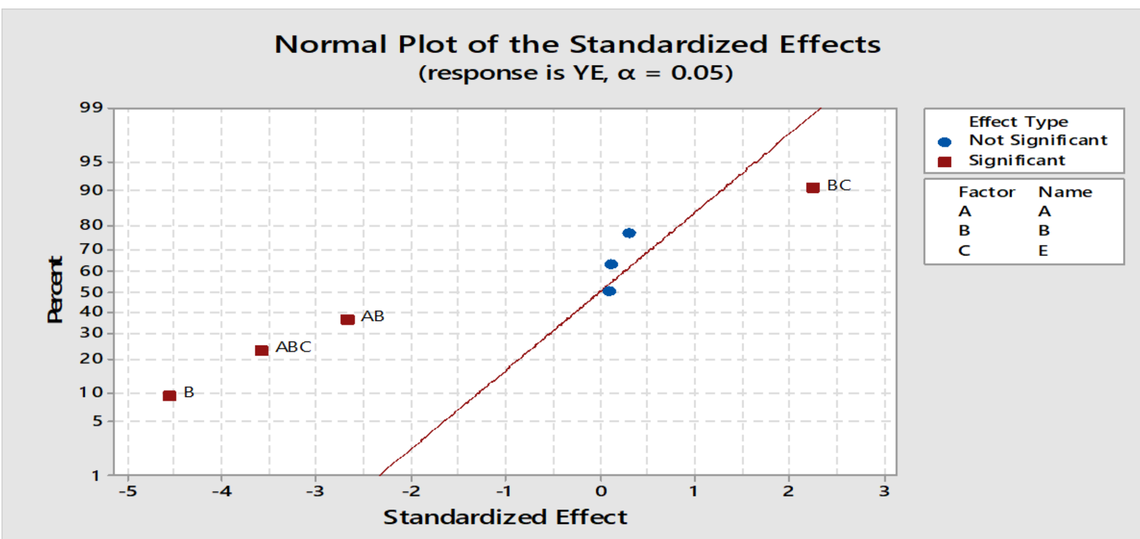

Figure 2: Normal probability plot for 3 - factors replicated factorial design

4. Discussion of results

This experiment utilized a 2^5 full factorial design with five factors (A, B, C, D, and E), each at two levels, and no replication. Table 2 shows the analysis of variance results for the unreplicated design, highlighting the inability to estimate experimental error due to the lack of replicates. The design table also confirms that all factors are orthogonal and free from aliasing. To address the limitation of a single replicate, a normal probability plot was constructed (Fig. 1). Effects that lie along the reference line are considered negligible, whereas those far from the line are deemed significant. The analysis revealed significant main effects for A, B, and E, as well as significant interactions involving AD, CD, DE, BCD, and ABCD. Factors C and D, along with their interactions, were found to be insignificant and were subsequently removed from the design. By eliminating C and D, the original 2^5 design was projected onto a 2^3 factorial design involving factors A, B, and E, with four replicates. The estimated effects indicated that A, B, and E had substantial influence. Furthermore, interaction plots indicated that factor D exhibited interactions with all other factors in the original design, even though D itself was not significant. Due to the absence of error terms, model diagnostics could not be performed on the original 25-run unreplicated design. However, Fig. 2 presents a new normal probability plot based on the projected 2^3 replicated design. This plot shows that factors A, E, and the AE interaction, were not significant, while factor B and the interactions AB, BE, and ABE emerged as significant. This research is not in consonant with (Wu and Hamada, 2009), which emphasized that transforming an unreplicated design into replicated one enhanced the ability to detect model accuracy. Also, the results of this work is in line with (Montgomery, 2019), which proposed that unreplicated designs can be augmented through foldover or point additions to introduce replication.

5. Conclusion

This study demonstrates that error variance cannot be estimated in a 2^5 unreplicated factorial design. By projecting the original design to a 2^3 replicated factorial design involving only the significant factors (A, B, and E), the model's accuracy improved. The fit and diagnostic checks for the replicated design yielded an accuracy of 55%, which is above average, underscoring the advantage of using replication in factorial designs. Therefore, as the number of factors (k) decreases in a projected factorial design, the number of feasible replicates increases, improving the reliability of statistical inference. Replicating factorial designs, even after projection, enhances precision and facilitates better model validation compared to unreplicated designs.

Declaration of Interest

The authors report there are no competing interest to disclose

References

Akra U. P., Bassey E. E., Etim A. C., Umondak U. J., Chiukpai A. I., and Akpan U. A. (2024). A Mixed-level Factorial Experiment to Elucidate the Impact of Inorganic Fertilizers on Crop Yield. *Research Journal of Agricultural Economics and Development*, 3(1); 61-73.

Akra, U. P., and Edet, E. B. (2017). Confounding 2^K Factorial Design to Obtain Optimal yields using Different Organic Manure. *Journal of Scientific and Engineering Research*, 4(11); 75-85.

Akra, U. P., Etim, A. C., Chiukpai, A. I., Umondak, U. J., and Inyang, E. E. (2025). A Multi- Level Factor Factorial Design for Solving Uncertainty Criteria in Establishing a Retail Business in Nigeria. *International Journal of Economic, Finance and Business Statistics*, 3(1), 1-14.

Akpan, S., Akra, U., Asuquo, B., and Udoeka, I. (2017). On a new L_P – Class to establish the relationship between various optimality criteria. *Global journal of Mathematics*, 10 (2), 656 – 664.

Carla, A., and Linda, L. H. (2016). Analysis of Augmented Unreplicated Factorial Designs Repeated in Time. *Quality and Reliability Engineering International Journal*, 32 (3), 855-862

Daniel, C. (1959). Use of half-normal plots in interpreting factorial two-level experiments, *Technometrics*, 1(5), 311-341.

Kenny, Y and Hamada, M. S. (2000). Critical Values of the Lenth Method for Unreplicated Factorial Designs. *Journal of Quality Technology*, 32(1):57-66.

Lawson, J. (2008). SAS macros for analysis of unreplicated 2*k* designs and 2*k* □ *p* designs with a possible outlier, *Journal of Statistical Software*, 25, 1-17.

Meixi, Y. (2014). Statistical Analysis of Unreplicated Factorial Designs Using Contrasts. Electronic Theses and Dissertations, digitalcommons.georgiasouthern.edu/etd/1147

Montgomery, D. C. (2019). Design and Analysis of Experiment (10th ed.). wiley

Tukey, J. (1949), One Degree of Freedom for Non-Additivity, Biometrics 5(3), 232-242.

Willers, J L., and Vinyard, B. T. (2019). A Classification of unreplicated factorial experiments for use with the analysis of deterministic simulation models. Conference on Applied Statistics in Agriculture, Kansas State University. 7(3), 124 - 137

Wu, C. F. J., and Hamada, M. (2009). Experiments: Planning, Analysis, and Optimization (2nd ed.). Wiley

Appendix 1
2^5 Unreplicated factorial design with randomized order

Std.Ord	Run.Ord	C.Pt	Blks	A	B	C	D	E	Y
8	1	1	1	1	1	1	-1	-1	7
28	2	1	1	1	1	-1	1	1	9
12	3	1	1	1	1	-1	1	-1	34
13	4	1	1	-1	-1	1	1	-1	55
19	5	1	1	-1	1	-1	-1	1	16
16	6	1	1	1	1	1	1	-1	20
29	7	1	1	-1	-1	1	1	1	40
17	8	1	1	-1	-1	-1	-1	1	60
25	9	1	1	-1	-1	-1	1	1	8
32	10	1	1	1	1	1	1	1	10
14	11	1	1	1	-1	1	1	-1	32
21	12	1	1	-1	-1	1	-1	1	50
30	13	1	1	1	-1	1	1	1	18
15	14	1	1	-1	1	1	1	-1	21
27	15	1	1	-1	1	-1	1	1	44
11	16	1	1	-1	1	-1	1	-1	61
31	17	1	1	-1	1	1	1	1	8
24	18	1	1	1	1	1	-1	1	12
22	19	1	1	1	-1	1	-1	1	35
10	20	1	1	1	-1	-1	1	-1	52
20	21	1	1	1	1	-1	-1	1	15
26	22	1	1	1	-1	-1	1	1	22
6	23	1	1	1	-1	1	-1	-1	45
7	24	1	1	-1	1	1	-1	-1	65
18	25	1	1	1	-1	-1	-1	1	6
2	26	1	1	1	-1	-1	-1	-1	10
5	27	1	1	-1	-1	1	-1	-1	30
23	28	1	1	-1	1	1	-1	1	53
3	29	1	1	-1	1	-1	-1	-1	15
4	30	1	1	1	1	-1	-1	-1	20
9	31	1	1	-1	-1	-1	1	-1	41
1	32	1	1	-1	-1	-1	-1	-1	63

• *Thank you for publishing with us.*